Global Correction of Magnetic Field Errors in LHC Interaction Regions

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Abstract

Global compensation of the field errors based on the minimization of nonlinearities in a one-turn map was found to be very effective in reducing the detrimental effects of magnetic field errors in the LHC during collision. With a few groups of low-order correctors, nonlinear terms in the one-turn map can be minimized order-by-order and, consequently, the dynamic aperture is substantially increased and the phase-space region occupied by beams becomes much more linear. One advantage of the global correction is the possibility of further optimization of the correction based on a direct measurement of a one-turn map with beam-dynamics experiments.

1 INTRODUCTION

During collisions, the dynamic aperture of the LHC is limited by the multipole field errors of superconducting high-gradient quadrupoles (MQX) of the inner triplets of the interaction regions (IRs). Control of these field errors is one of the primary tasks in the design of the LHC IRs. With the current reference harmonics of Fermilab and KEK MQXs [1], correctors are necessary for the IRs in order to meet the dynamic aperture requirement of the LHC. Because of the beam separation in the triplets due to an angle crossing of colliding beams, high-order multipoles of the field errors feed down to low-order nonlinearities of the system and they are important to the aperture limitation. It is, however, difficult to correct those high-order multipoles errors by using the traditional methods of local correction since it is difficult and costly to build high-order multipole correctors. The global correction of magnetic field errors based on the minimization of the nonlinearities in a Poincaré map of a circular accelerator is an alternative way to reduce the detrimental effects of both the systematic and random field errors [2]. In a circular accelerator, the nonlinear beam dynamics can be described by a Poincaré map known as one-turn map. The one-turn map contains all global information of nonlinearities in the system. By minimizing the nonlinear terms of a one-turn map order-by-order with a few groups of multipole correctors, one can reduce the nonlinearity of the system globally [2]. In this paper, the effectiveness and feasibility of the global correction of the magnetic field errors in the triplets of IRs is investigated for the LHC collision lattice. It was found that the global correction strategy is effective and efficient in increase of the dynamic aperture and improvement of the linearity of the phase-space region occupied by beams for the LHC during collisions. One advantage of the global correction of nonlinear fields is that the correction may be further optimized during the commission of an accelerator based on measurements of a one-turn map in beam-dynamics experiments. Methods for a direct measurement of a one-turn map with beam-dynamics experiment has recently been proposed and technique problems associated with such a measurement has been studied in detail [3, 4, 5, 6].

This paper is organized as follows. In Section 2, we discuss the principle of global correction of nonlinear fields. In Section 3, the test lattice for the LHC during collisions is presented. In Sections 4, the effectiveness of the global correction on the improvement of the dynamic aperture and the improvement of the linearity of the phase space are studied. In Section 5, we discuss the robustness of the global correction. Section 6 contains a conclusion.

2 GLOBAL COMPENSATION OF THE NONLINEAR FIELDS

Neglecting the coupling between the transverse and longitude motion, at any “check-point” of an accelerator, the transverse motion of beam particles can be described mathematically by a 4-dimensional one-turn map in the form of Taylor expansion

$$\vec{Z}\,' = \mathcal{M}\vec{Z} = \sum_{n=1} \left(\sum_{i+j+k+l=n} \vec{u}_{ijkl} \;\; \xi_x^i \; \eta_x^j \; \xi_y^k \; \eta_y^l \right) \quad (1)$$

where $\vec{Z} = (\xi_x, \eta_x, \xi_y, \eta_y)$ is the normalized phase-space vector and $\eta_{x,y}$ are the conjugate momenta of $\xi_{x,y}$. $\vec{Z} = 0$ is the closed orbit and $\vec{u}_{ijkl}$ are constant coefficients containing all global information of nonlinearities of the system. If the close orbit is at the center of magnets, the nth-order terms of a one-turn map are the contributions from the multipole components of the error fields with order up to n. On the other hand, if the close orbit is not at the center of magnets due to magnet misalignments or beam crossing at interaction points, high-order multipole errors feed down to low-order terms of the one-turn map and, consequently, $\vec{u}_{ijkl}$ of order n are functions of all multipole components. For an accelerator, since the phase-space region near the origin is of most interest, low-order terms of a one-turn map are usually more important than high-order terms of the map. The low-order multipole components of error fields are therefore important to the beam dynamics. Because of the feed-down effect, however, the high-order multipole errors contribute also to low-order terms of the map and become important to the beam dynamics as well. The global correction of the nonlinearities is based on an

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assumption that with a few groups of multipole correctors, $\vec{u}_{ijkl}$ with $i+j+k+l \geq 2$ can be minimized order-by-order and, consequently, the nonlinearities of the system can be substantially reduced. In order to minimize undesirable $\vec{u}_{ijkl}$ with a few parameters of the correctors, we postulate that the nth-order undesirable nonlinearity in a one-turn map can be characterized by the magnitude of its nth-order undesirable coefficients which are defined by

$$\lambda_2 = \left(\sum_{i+j+k+l=2} \left| \vec{u}_{ijkl} - \vec{u}^0_{ijkl} \right|^2 \right)^{1/2} \tag{2}$$

and

$$\lambda_n = \left(\sum_{i+j+k+l=n} \left| \vec{u}_{ijkl} \right|^2 \right)^{1/2} \qquad \text{for } n > 2, \tag{3}$$

where $\vec{u}^0_{ijkl}$ of $i+j+k+l=2$ denote the quadratic terms contributed by sextupole chromaticity correctors. To minimize the undesirable nonlinearities, the quadratic nonlinearity for the chromaticity correction needs to be subtracted from the $\vec{u}_{ijkl}$. For convenience, we define the nth-order global correction when all λ_i with $i = 2, ..., n$ are minimized order-by-order using the multipole correctors up to the nth order. For example, for the 2nd-order global correction λ_2 of quadratic terms of a one-turn map will be minimized by using sextupole correctors and for the 3rd-order global correction both λ_2 and λ_3 will be minimized by using sextupole and octopole correctors. To implement the global correction of the nonlinear fields during design and construction of an accelerator, the one-turn map is obtained by using the method of Lie algebra [7] or automatic differentiation (differential algebra) [8] with measured magnetic field errors. During the commission of an accelerator, the global correction of the nonlinear fields may be further optimized if a one-turn map can be extracted with desired accuracy directly from beam dynamics measurements. Such a beam-based global correction needs only a measurement of low-order map since the study showed [2] that the low-order global correction is usually sufficient even in the case that the high-order multipole errors are important.

To illustrate this minimization procedure, let us consider four global correctors of the nth-order multipole for minimizing the nth-order nonlinear terms of the map. Consider the situation that these correctors are installed at locations where the closed orbit is at the center of the correctors. Suppose that a one-turn map is measured at a "check-point" between the 1st and 4th corrector. Let $\exp\left\{ : C^{(i)}_{n+1}\left(\vec{Z}\right) : \right\}$ be the Lie transformation for the ith corrector, where $i = 1, ..., 4$ and $C^{(i)}_{n+1}$ is a homogeneous polynomial of $\vec{Z}$ in degree $n+1$; $\mathcal{M}_i$ be the transfer map between two adjacent correctors when $i = 1, 2, 3$ and between the "check-point" and the adjacent correctors when $i = 0, 4$; and

$$\mathcal{M}_{i4} = \prod_{k=i}^{4} \mathcal{M}_k, \tag{4}$$

where $\mathcal{M}_{04}$ is the one-turn map of the ring without the nth-order correctors. The one-turn map of the ring with the nth-order correctors can be written as

$$\mathcal{M} = \mathcal{M}_{04} \prod_{i=1}^{4} \exp\left\{ : C^{(i)}_{n+1}\left(\mathcal{M}^{-1}_{i4} \vec{Z} : \right) \right\}. \tag{5}$$

Let $\mathcal{R}_{i4}$ be the linear transfer matrix associated with $\mathcal{M}_{i4}$. Then

$$\mathcal{M}^{-1}_{i4} \vec{Z} = \mathcal{R}^{-1}_{i4} \vec{Z} + \sigma_2(\vec{Z}), \tag{6}$$

where $\sigma_{k+1}(\vec{Z})$ represents a remainder series consisting of terms higher than the kth-order, and

$$C^{(i)}_{n+1}\left(\mathcal{M}^{-1}_{i4} \vec{Z} \right) = C^{(i)}_{n+1}\left(\mathcal{R}^{-1}_{i4} \vec{Z} \right) + \sigma_{n+2}(\vec{Z}). \tag{7}$$

It should be noted that Eq. (7) is valid only when the closed orbit is at the center of the correctors, otherwise, terms lower than the $(n+1)$th-order are also involved. Since the lowest-order terms in the remainder series $\sigma_{n+2}(\vec{Z})$ are the $(n+2)$th-order, for the minimization of the nth-order terms, $\sigma_{n+2}(\vec{Z})$ can be neglected and

$$\mathcal{M} \simeq \mathcal{M}_{04} \prod_{i=1}^{4} \exp\left\{ : C^{(i)}_{n+1}\left(\mathcal{R}^{-1}_{i4} \vec{Z} \right) : \right\} \tag{8}$$

where $\mathcal{M}_{04}$, the one-turn map without the nth-order correctors, and $\mathcal{R}_{i4}$, the linear transfer matrices, can be either calculated based on the design lattice and the measured field errors or directly measured from beam-dynamics experiments. By using Eq. (8), the nth-order nonlinearity of $\mathcal{M}$ can then be minimized by adjusting the nth-order correctors $C^{(i)}_{n+1}$. It should be noted that for the beam-based global correction, only one measurement of $\mathcal{M}_{04}$ is required for the minimization of λ_n.

3 THE TEST LATTICE

The test lattice used in this study is the LHC version 5.0. The LHC has four interaction regions (IRs): IR1 and IR5 are high luminosity interaction points ($\beta^* = 0.5$ m) and IR2 and IR8 low luminosity points. The layout of the inner triplets of four IRs is almost identical. Each inner triplet comprises four superconducting high gradient quadrupoles (MQX), Q1, Q2A, Q2B, and Q3. Due to the beam separation and the large β_{max}, the beam dynamics during collisions is dominated by the field errors of MQX. In this study we therefore consider only the field errors of MQX. The random multipole components of MQX are chosen with Gaussian distributions centered at zero and truncated at $\pm 3\sigma_{b_{n+1}}$ or $\pm 3\sigma_{a_{n+1}}$ where $\sigma_{b_{n+1}}$ and $\sigma_{a_{n+1}}$ are the rms value of the nth-order normal and skew multipole coefficient, respectively. Reference harmonics of version 2.0 is used in this study for both Fermilab and KEK MQX [1]. The uncertainty of a systematic error is simply added to the systematic error in such a way that it maximizes the systematic error. Due to the consideration of a larger systematic b_{10} in KEK quadrupoles, two different arrangement of MQX, mixed and unmixed configuration, are studied. In the unmixed configuration, the Fermilab MQX are installed in the triplets of IP1 and IP2, and the KEK MQX in the triplets of IP5 and IP8. In the mixed configuration, four MQX in each triplet are mixed with two quadrupoles from Fermilab and another two from KEK. In this case, the Fermilab MQX are installed at Q2A and Q2B and KEK MQX at Q1 and Q3. Since the β_{max} ($\sim$ 4700 m) in the triplets of IP1 and IP5 is more than 10 time larger that that of IP2 and IP8, the field quality in the triplets of IP1 and IP5 is far more important than that of IP2 and IP8. To compensate the error fields in the triplets of IP1 and IP5, each triplet contains three corrector packages. In this study, we use four groups of correctors, one in each triplet of IP1 and IP5, to minimize λ_n order-by-order. To test the global nature of the correction, we also include four corrector packages outside the triplets to corrector the nonlinear fields in the triplets. Each package of the corrector contains normal and skew components of a desired multipole corrector. It was found that in the sense of improvement of the dynamic aperture, the correctors outside the triplets is as effective as the correctors in the triplets for the global correction of the field errors in the triplets [2]. In this study, the crossing angle of two counter-rotating beams is taken to be 300 μrad. The fractional parts of horizontal and vertical tunes are $\nu_x = 0.31$ and $\nu_y = 0.32$, respectively.

4 EFFECT OF THE GLOBAL CORRECTION OF NONLINEAR FIELDS

To study the effect of the global correction of nonlinear fields, dynamic aperture (DA) of the system are calculated before and after the correction. In order to reduce the sensitivity of the DA to the choice of initial launch point for tracking in phase space, we define an aperture as the shortest distance from the origin in the four-dimensional normalized phase space during the tracking. To find the DA, the launch point is moved away from the origin until the particle is lost. No physical aperture limit is imposed in the ring and a particle is defined to be lost if $x^2 + y^2 \geq (10\ \mathrm{cm})^2$ where x and y are its horizontal and vertical coordinates, respectively. The DA defined in this manner is found to be relatively insensitive to the choice of launch point in phase space. Tracking of particle motions has been done without synchrotron oscillations and momentum deviations. The DA has been calculated with 10^5-turn tracking. To improve the statistical significance of the simulations, we have used 50 different samples of random multiple components generated with different seed numbers in a random number generator routine.

Figures 1 and 2 are the DA of 50 random samples with or without the global correction of the nonlinear fields for the unmixed and mixed configuration, respectively. Without any correction (Figs. 1a and 2a), the smallest and the average DA of 50 samples is found to be 5.5σ and 7.9σ for the unmixed configuration and 6.5σ and 8.0σ for the mixed

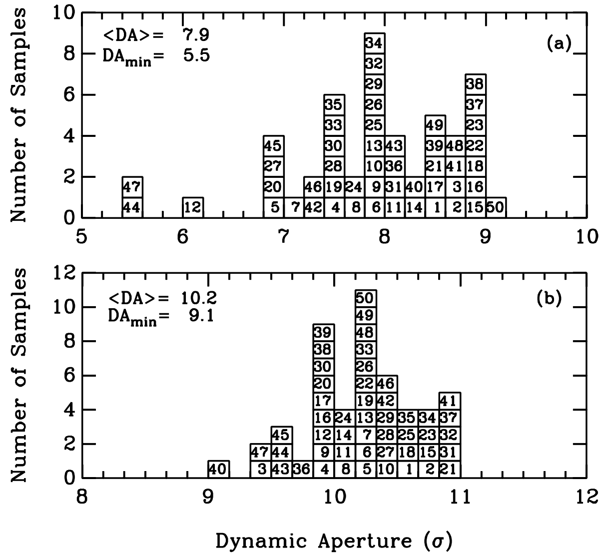

Figure 1: Dynamic aperture of fifty samples of the LHC collision lattice with the unmixed configuration. (a) without any correction for the nonlinear fields; (b) with the 3rd-order global correction for the nonlinear fields using four sextupole and octopole correctors. The number in each block identifies each sample.

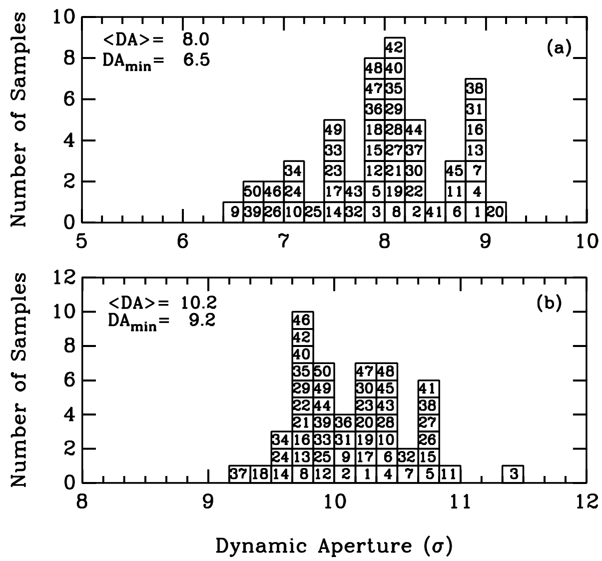

Figure 2: The same as in Fig. 1 but with the mixed configuration.

configuration, respectively, where σ is the transverse beam size. At the high luminosity IPs, $\sigma = 15.9\mu$m. A smaller DA for the unmixed configuration is due to a larger b_{10} in KEK quadrupoles. After the 3rd-order global compensation with sextupole and octopole correctors outside the triplets (Figs. 1b and 2b), the smallest and the average DA increases to 9σ and 10σ for both configurations. It should be noted that with the conventional (local) correction of the field errors, high-order correctors have to be used in order to achieve a significant improvement in the DA [9].

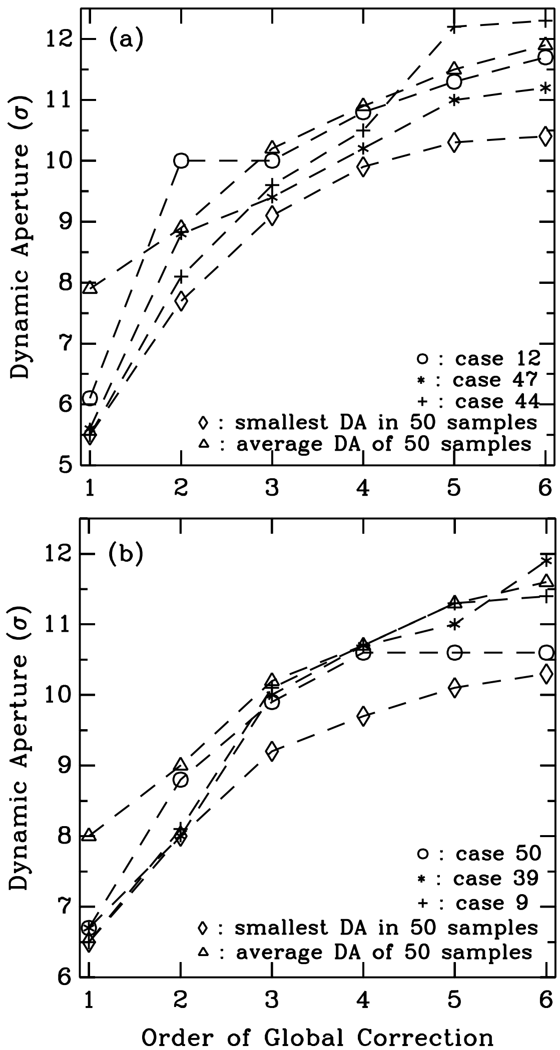

Figure 3: The DA after the global correction vs. the order of the correction. $n = 1$ indicates the cases without the correction. (a) The unmixed configuration. Case 12, 47, and 44 are three worst cases without the correction. (b) The mixed configuration. Case 9, 39, and 50 are three worst cases without the correction.

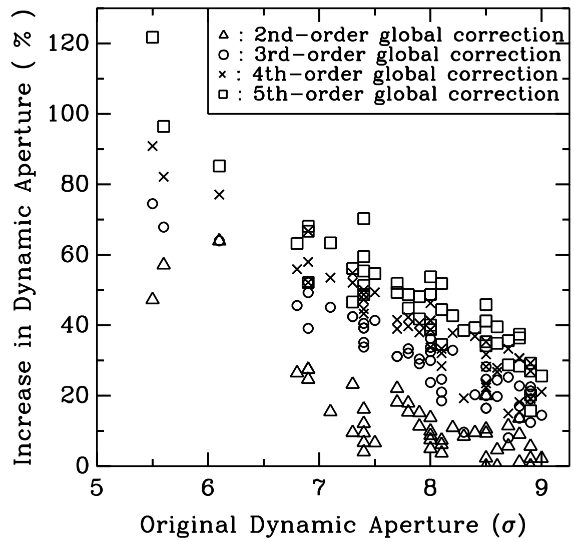

Figure 4: The increase of the DA after the global correction vs. the DA without the correction for the fifty samples of the unmixed configuration.

Because of the beam separation in the triplets, high-order multipoles of the field errors feed down to low-order terms of the one-turn map so that they are important to the DA. In the conventional correction, the field errors are compensated locally based on the errors of each magnets and, therefore, the high-order correctors have to be used in order to reduce the effects of high-order multipoles. For the global correction of the field errors, on the other hand, a few sextupole correctors can minimize the dominant nonlinear terms, quadratic and cubic terms, of the map and achieve a significant reduction of the nonlinearity of the system.

Fig. 3 plots the DA after the global correction as a function of the order of the correction. It shows that as the order increases the further improvement of the DA becomes less pronounced, which indicates that the lower-order (quadratic and cubic) nonlinear terms of the one-turn map dominates the nonlinear dynamics of the system. In Fig. 4, the percentage increase of the DA after the global correction is plotted $vs.$ the original DA without any correction. In general, the smaller the original DA, the larger the increase of the DA after the correction. For example, without any correction, two worst cases of the unmixed configuration, case 44 and 47, have a DA smaller than 6σ (see Fig. 1a). After the 2nd-order correction, the DA gains about 50% for both cases. After the 3rd-order correction, the DA becomes larger than 9σ for both cases, which is a more than 60% gain in DA. As the original DA increases, the gain of the DA after the global correction diminishes. It is understandable that if the original system is already quite linear, the correction of the nonlinear fields will not result in a substantial improvement.

A strong nonlinearity in the lattice can lead to a substantial degree of amplitude dependence of betatron tunes even in a phase-space region near the origin, and this may result in crossings of dangerous resonances and a reduction in the dynamic aperture. Minimizing the amplitude dependence of tunes is thus desirable for a stable operation of an accelerator. Previous studies [10, 11, 12] showed that both the local correction for the systematic field errors and the sorting of magnets for the random field errors are effective in reducing the amplitude dependence of tunes. The effect of the global correction of the nonlinear fields on the amplitude dependence of tunes are also studied by using the method of normal form. In Figs. 5 and 6, the detuning functions, $\delta\nu_x$ and $\delta\nu_y$, for case 44 of the unmixed configuration are plotted as functions of the action variables I_x and I_y, respectively, where $\delta\nu_x$ and $\delta\nu_y$ are calculated at IP1. Without any correction, both horizontal and vertical tune strongly depend on I_x and I_y as shown in Figs. 5a and 6a. Figs. 5b and 6b show the nonlinear tune shifts after the 3rd-order global correction. A comparison between

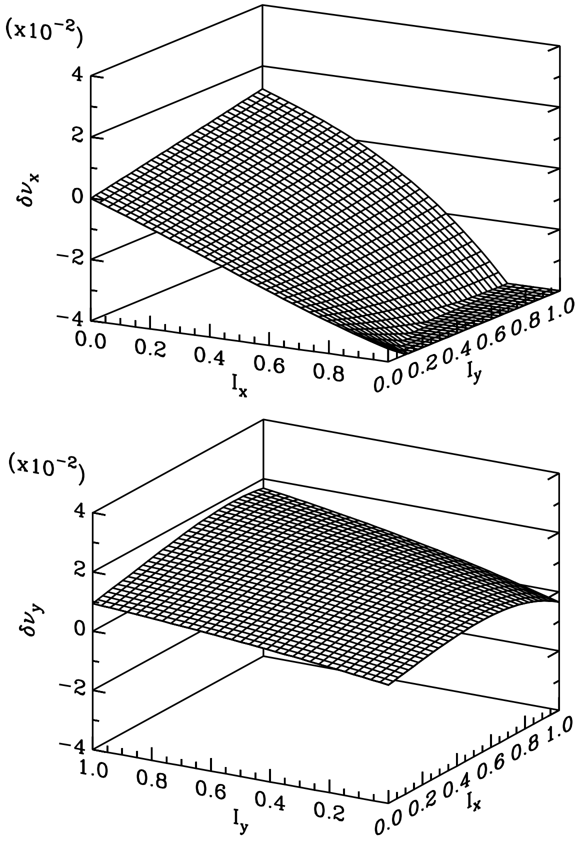

Figure 5: Amplitude dependence of tunes of case 44 of the unmixed configuration without any correction. $\delta\nu_x$ and $\delta\nu_y$ are calculated at IP1. The unit of I_x and I_y is 10^{-8} m. At IP1, $I_x + I_y = 10^{-8}$ m corresponds to $\sim 6\sigma$.

the uncorrected and corrected lattice shows that the global correction effectively suppresses the nonlinear tune shift. Other cases have a similar situation.

The improvement of linearity of the phase-space region near the origin can also be directly examined with phase-space plots. Figs. 7 and 8 are the phase-space plots of case 44 of the unmixed configuration before and after the global correction, which shows that the phase-space region occupied by the beams becomes much linear after the global compensation of the field errors even in the case that only four sextupole correctors are used. It should be noted that the dynamic aperture calculated from the tracking of 10^5 turns does not really tell the performance when the storage time of at least several hours is in question. However, by examining the linearity of phase space together with the amplitude dependence of tunes, one may get a better idea of the long-term storage performance.

It should be noted that even though the results reported in this section are all for the working point of $\nu_x = 0.31$ and $\nu_x = 0.32$, the effectiveness of the global compensation has also been demonstrated on the LHC lattice with other working points.

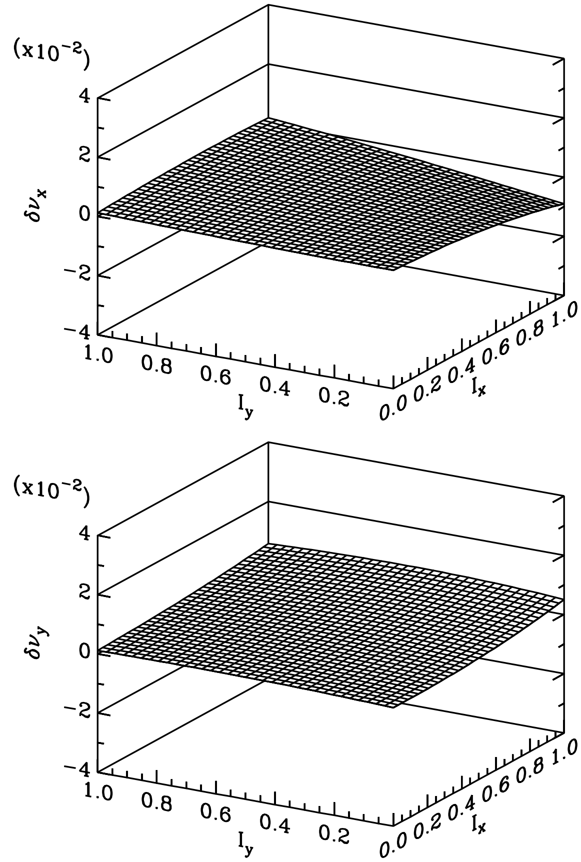

Figure 6: The same as Fig. 5 but with the 3rd-order global correction.

5 ROBUSTNESS OF THE GLOBAL CORRECTION OF NONLINEAR FIELDS

The use of the global correction requires the knowledge of a one-turn map. A one-turn map, either calculated based on the design lattice and the measured field errors or measured directly from beam-dynamics experiments, always contains errors or uncertainty. The sensitivity of the global correction to the uncertainty in the map is important to the feasibility of the global correction scheme. The uncertainty in the map can be divided into two parts, the uncertainty in linear transfer matrices and the uncertainty in nonlinear terms of the map. The former is mainly due to the lack of knowledge on the linear lattice and the latter due to both the uncertainty of linear lattice and the errors in the multipole measurement or the measurement errors in beam-dynamics experiments. Previously, the global correction was found to be not very sensitive to the uncertainty in the nonlinear terms of the map [2]. Since the global correctors may not be close to the sources of nonlinear fields, the uncertainty in the linear transfer matrices, on the other hand, could make the global correction ineffective. To investigate the effect of the uncertainty in the linear transfer matrices $\mathcal{R}_{i4}$, we

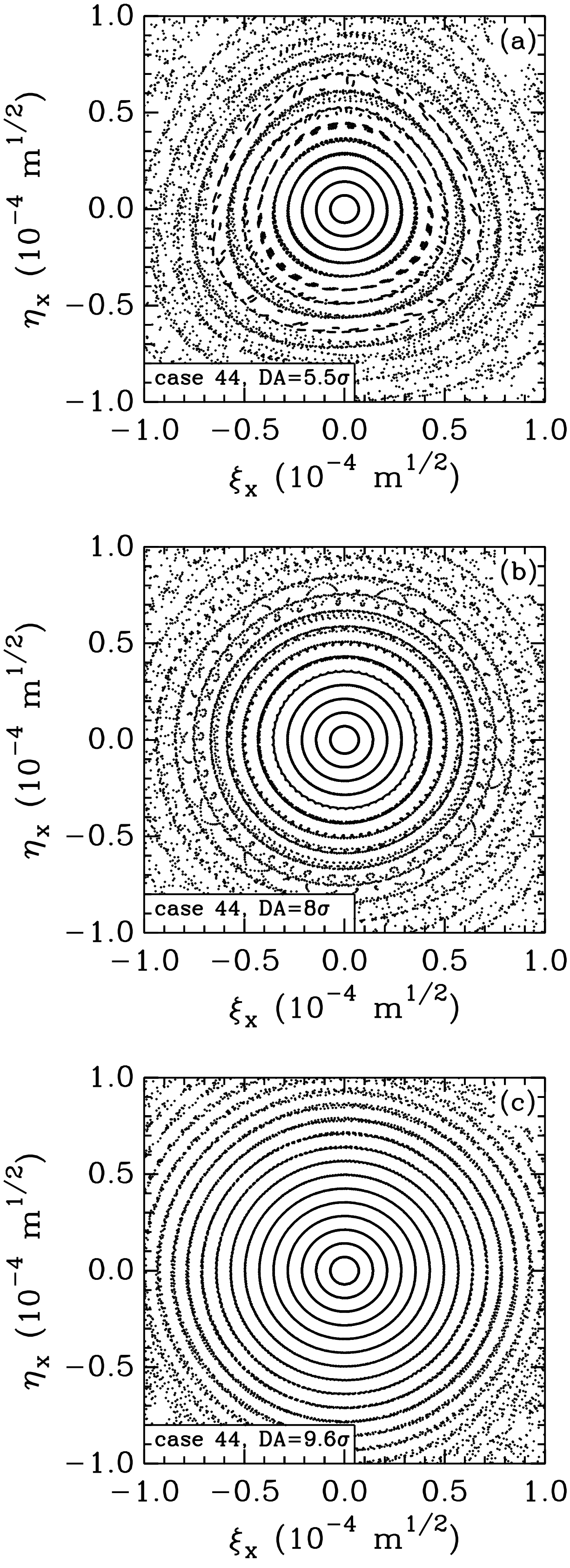

Figure 7: Normalized phase-space plot on the horizontal plane at IP1 for case 44 of the unmixed configuration. (a) without any correction; (b) with the 2nd-order global correction; and (c) with the 3rd-order global correction.

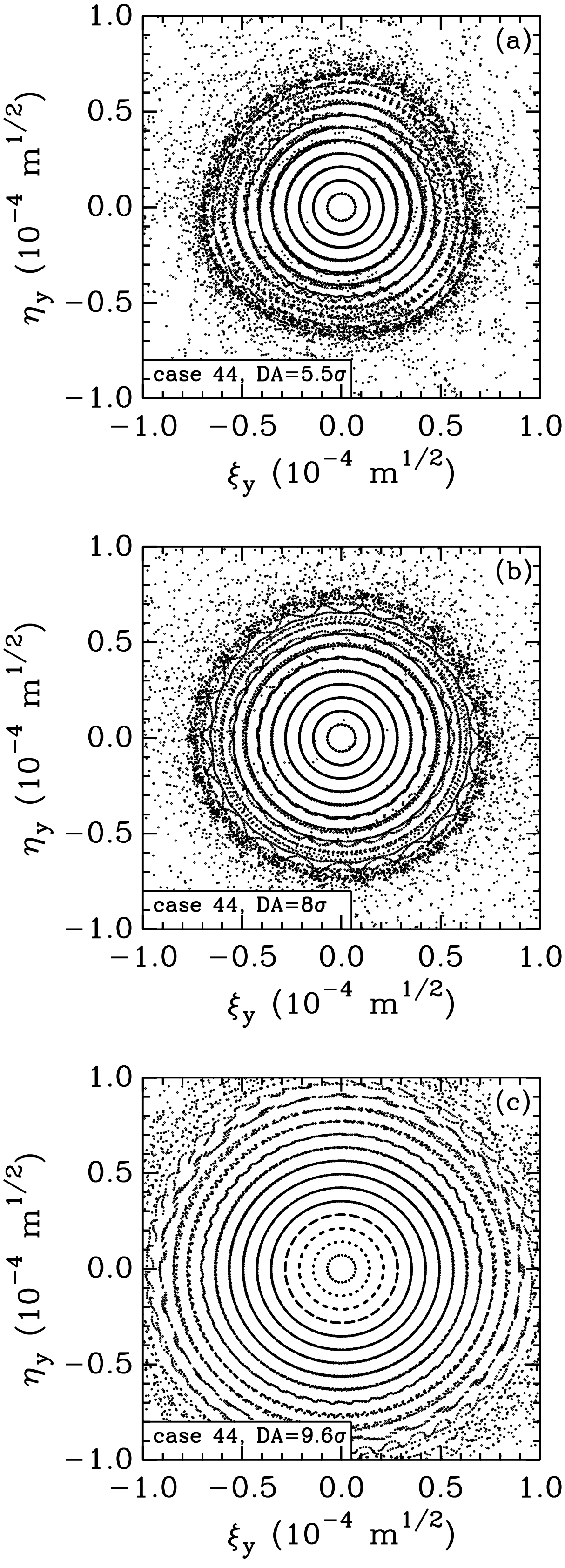

Figure 8: The same as Fig. 7 but for normalized phase-space plot on the vertical plane.

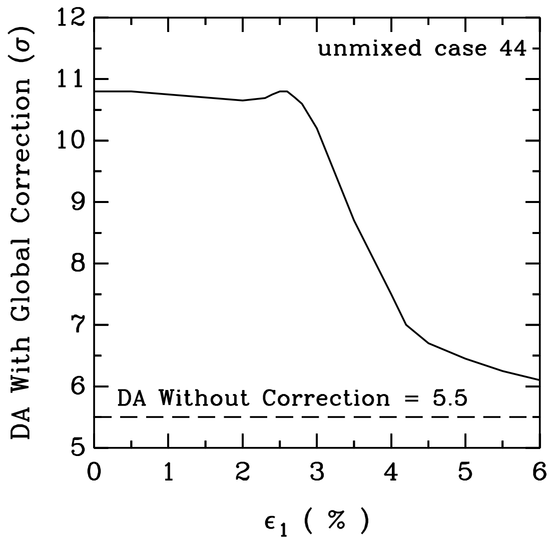

Figure 9: The DA after the 4th-order global correction vs. the uncertainty in linear transfer matrices ϵ_1 for case 44 of the unmixed configuration.

assume that the error of matrix element r_{lk} of $\mathcal{R}_{i4}$ is

$$\delta r_{lk} = \epsilon_1 f r_{lk} \tag{9}$$

where ϵ_1 is the maximal percentage of errors in matrix elements of $\mathcal{R}_{i4}$ and f is a random number in [-1, 1]. Fig. 9 plots the DA after the 4th-order global correction as a function of ϵ for case 44 of the unmixed configuration, which shows that uncertainty of 3% or less in linear transfer matrices have little impact on the global correction, but uncertainty of 5% or more can make the global correction ineffective. It should be noted that a measurement of the linear transfer matrices with better then 3% uncertainty is achievable when the measurement system is well debugged. Moreover, since the global correctors can be adjusted during operation of an accelerator, the global correction can be fine tuned when the knowledge of the linear lattice is improved.

6 CONCLUSIONS

The global correction of magnetic field errors based on the minimization of nonlinearities in a one-turn map is an effective means to suppress the detrimental effects of systematic as well as random field errors in the LHC during collisions. With a few groups of multipoles correctors, nonlinear terms in a one-turn map can be minimized order-by-order and, consequently, the nonlinearity of the system is significantly reduced which results in an increase of the dynamic aperture and improvement of the linearity of the phase-space region occupied by beams. Compared with the local corrections of the field errors, the global correction has several advantages. (a) The random field errors of large number of magnets can be compensated with a few groups of independent powered correctors. (b) Since the low-order nonlinear (quadratic and cubic) terms of the map usually dominate the beam dynamics, only low-order (sextupole and octopole) correctors are needed for the global correction even though high-order multipoles are important to the beam dynamics due to the feed-down effect. (c) The global correction of the nonlinear fields may be further optimized with a direct measurements of a one-turn map in beam-dynamics experiments. This beam-based correction is especially important when there is a significant uncertainty in the field measurement of magnets or a significant change of the field errors during the operation of a superconducting ring. While the global correction of the field errors partially suppresses the low-order nonlinear effects of the random and systematic errors, the local corrections of the field errors, on the other hand, can effectively compensate low-order systematic errors to a large extent. It is, therefore, important to stress that the global correction of the field errors should never be considered as "cure-all" in dealing with the nonlinearity in superconducting magnets and it should be regarded as a complement to the local correction of the field errors.

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